Growing Sequences

Arithmetic Sequence	Common Difference
Geometric Sequence	Common Ratio
Initial Term	

Problem Situation: The Brown Tree Snake

The Brown Tree Snake is responsible for entirely wiping out over half of Guam's native bird and lizard species as well as two out of three of Guam's native bat species. The Brown Tree Snake was inadvertently introduced to Guam by the US military due to the fact that Guam is a hub for commercial and military shipments in the tropical western Pacific. It will eat frogs, lizards, small mammals, birds and birds' eggs, which is why Guam's bird, lizard, and bat population has been affected. Listed in the table below is the data collected on the Brown Tree Snake's invasion of Guam.

Adapted from Global Invasive Species Database (http://www.issg.org/database/species/ecology.asp?si=54)

The number of snakes for the first few years is summarized by the following sequence:

1, 5, 25, 125, 625, . . .

- What are the next three terms of the sequence?
- How did you predict the number of snakes for the 6th, 7th, and 8th terms?
- What is the initial term of the sequence?
- What is the pattern of change?
- Do you think the sequence above is an arithmetic sequence? Why or why not?

Growing Sequences: Review of Arithmetic Sequences

An **arithmetic sequence** goes from one term to the next by always adding (or subtracting) the same value. For instance, 2, 5, 8, 11, 14, . . . and 7, 3, –1, –5, . . . are arithmetic, since you add 3 in the first sequence and subtract 4 in the second sequence, respectively, at each step.

The number added (or subtracted) at each stage of an arithmetic sequence is called the **common difference** d, because if you subtract (find the difference of) successive terms, you'll always get this common value.

For example, find the common difference and the next term of the following sequence:

3, 11, 19, 27, 35, . . .

To find the common difference, I have to subtract a pair of terms. It doesn't matter which pair I pick, as long as they're right next to each other:

11 – 3 = 8
19 – 11 = 8
27 – 19 = 8
35 – 27 = 8

The difference is always 8, so ***d* = 8**. Then the next term is 35 + 8 = **43**.

For arithmetic sequences, the common difference is d, and the first term a_1 is often referred to as the **initial term** of the sequence. In the Brown Tree Snake sequence, the rate of change is not arithmetic as shown below.

1, 5, 25, 125, 625, . . .

5 – 1 = 4
25 – 5 = 20
125 – 25 = 100
635 – 125 = 500

The difference is not a common number; therefore, the sequence is not arithmetic. So, what kind of sequence is this? Strangely enough, the pattern that I see is one of multiplication.

1, 5, 25, 125, 625, . . .

1 • 5 = 5
5 • 5 = 25
25 • 5 = 125
125 • 5 = 625

The initial term of the Brown Tree Snake is 1 and the rate of change is that of multiplication by 5 each time in order to generate the next terms of the sequence. This type of sequence is called a geometric sequence. A **geometric sequence** goes from one term to the next by always multiplying (or dividing) by the same value. So 1, 2, 4, 8, 16, . . . and 81, 27, 9, 3, 1, 1/3, . . . are geometric, since you multiply by 2 in the first sequence and divide by 3 in the second sequence, respectively, at each step.

The number multiplied (or divided) at each stage of a geometric sequence is called the **common ratio *r***, because if you divide successive terms, you'll always get this common value. So, let's determine the common ratio *r* of the Brown Tree Snake Sequence.

1, 5, 25, 125, 625, . . .

5/1 = 5
25/5 = 5
125 /5 = 5
625/5 = 5

The common ratio of the Brown Tree Snake is $r = 5$. Let's now find the initial term and the common ratio of other geometric sequences.

Example 1: 1/2, 1, 2, 4, 8, . . .

Initial term:________ Common ratio:________

Example 2: 2/9, 2/3, 2, 6, 18, . . .

Initial term:________ Common ratio:________

Now it is time for you to determine if the following sequences are arithmetic or geometric. On the next page, you will find some problems to help you practice your skills on sequences.

Information adapted from The Purple Math Forums (http://www.purplemath.com/modules/series3.htm)

Practice with Sequences

For a sequence, write arithmetic and the common difference or geometric and the common ratio. If a sequence is neither arithmetic nor geometric, write neither.

1) 2, 6, 18, 54, 162, ... _______________________ common ___________ = _____
2) 14, 34, 54, 74, 94, ... _______________________ common ___________ = _____
3) 4, 16, 36, 64, 100, ... _______________________ common ___________ = _____
4) 9, 109, 209, 309, 409, ... _______________________ common ___________ = _____
5) 1, 3, 9, 27, 81, ... _______________________ common ___________ = _____

Given the initial term and either common difference or common ratio, write the first 6 terms of the sequence.

6) $a_1 = 7, r = 2$ ____
7) $a_1 = 7, d = 2$ ____
8) $a_1 = 3, r = 5$ ____
9) $a_1 = 4, d = 15$ ____

The water hyacinth is an invasive species from Brazil, which has found its way into North Carolina in the north and inland of the Tar and Neuse river areas. Unchecked, the water hyacinth can lead to clogged waterways, altered water temperature and chemistry, and the exclusion of native plants and wildlife in our own state. Some NC biologists found a region in which 76.9 $miles^2$ were covered by the water hyacinth. They decided to monitor the area by checking it again every 10 days. Here's the data that they collected:

76.9; 157.8; 315.6; 631.2; 1,262.4, . . .

10) Is the area of the plant growing arithmetically or exponentially? Explain how you know by listing the features of the sequence (common difference or common ratio).

11) How is this problem different from the Brown Tree Snake sequence?