The modeling of time-distance relationships is a very useful application of algebra. In this investigation, you will explore time-distance relationships by considering various walking scenarios. You will learn how the starting position, speed, direction, and final position of a walker influence the graph and the equation of a time-distance relationship. We will be using motion detectors to measure distance walked over time. Let's consider the two graphs below:


Who started further away from the motion detector? How can you tell?

Who is walking faster? How can you tell?

Fill in the table below.

|  | Steve | Jamal |
| :---: | :---: | :---: |
| Starting Position |  |  |
| Total Time |  |  |
| Speed |  |  |
| Direction |  |  |
| Final Position |  |  |

adapted from Discovering Algebra: An Investigative Approach by Murdock, Kamischke, \& Kamischke

Below you will find a set of walking instructions. Sketch the graph produced by each one. Take turns being the walker!
A: Stand 1 meter from the motion detector. Walk
C: Stand 2 meters from the motion detector. Stand away at a steady pace.


B: Stand 4 meters from the motion detector. Walk towards the motion detector at a steady pace.

still for 2 seconds, then walk away at a steady pace.


D: Stand 1.5 meters from the motion detector. Walk away slowly at first, then speed up.


Try to replicate the shape of each of the graphs below. Take turns being the walker! For each graph, write a set of walking instructions. Tell where the walk begins, how fast the person walks (in $\mathrm{m} / \mathrm{s}$ ), and whether the person walks toward or away from the motion detector.


Without using the motion detector, draw the graph of a 6 -second walk based on each set of walking instructions or data.

1) Start at the 2.5 -meter mark and stand still.
2) Start at the 3-meter mark and walk toward the sensor at a constant rate of 0.4 meter per second.
3) 

| Time $(\mathrm{s})$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(\mathrm{m})$ | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |



Apply the Math: Let's see if you've got it!
The time-distance graph shows Carol walking at a steady rate. Her friend Bobby used a motion detector to measure her distance over time. Answer the questions about the graph.

1) According to the graph, how much time did Carol spend walking?
2) Was Carol walking toward or away from the motion detector? Explain your thinking.
3) Approximately how far away from the motion detector was she when she started walking?
4) If you know Carol is 2.9 m away from the motion detector after 4 s , how fast was she walking?

5) If the equipment will measure distances only up to 6 m , how many more seconds of data can be collected if Carol continues walking at the same rate?
6) Looking only at the graph, how do you know that Carol was neither speeding up nor slowing down during her walk?

Follow up problems:

Graph a walk for the following set of instructions: "Start at the 0.5 -meter mark and walk away from the motion detector at a steady pace of 0.25 meter per second for 5 seconds."

Write a set of walking instructions based on the table data and sketch a graph of the walk.

| Time $(\mathrm{s})$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance $(\mathrm{m})$ | 4.0 | 3.6 | 3.2 | 2.8 | 2.4 | 2.0 | 1.6 |

Write a set of walking instructions for this graph:


At what rate in $\mathrm{ft} / \mathrm{s}$ would you walk so that you were moving at a constant speed of 1 mph ?

Describe how the rate of walking affects the graph of each situation:
a) The graph of a person walking toward a motion detector at a constant rate.
b) The graph of a person standing still.
c) The graph of a person walking slowly away from the motion detector.

Describe how you would tell someone to walk the line $y=x$, where $x$ is measured in seconds and $y$ is measured in feet. Describe how to walk the same line where $x$ is measured in seconds and $y$ is measured in meters. Which represents a faster rate? Explain.

For each situation, determine if it is possible to collect such walking data and either describe how to collect it or explain why it is not possible.

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Samantha's walk was recorded by a motion detector. A graph of her walk and a few data points are shown below.
a) Write an equation in the form distance from motion detector $=$ start distance + change to model this walk.
b) If she continues to walk at a constant rate, at what time would she pass the motion detector?

| Time <br> $(\mathrm{s})$ | Distance <br> $(\mathrm{m})$ |
| :---: | :---: |
| 0 | 3.5 |
| 2 | 3 |
| 6 | 2 |



Match the walking instructions with their graph sketches.
i.

ii.

iii.

a) The walker stands still.
b) The walker takes a few steps towards the motion detector, then walks away.
c) The walker steps away from the motion detector, stops, then continues more slowly in the same direction.

Andrei and his younger brother are having a race. Because the younger brother can't run as fast, Andrei lets him start out 5 m ahead. Andrei runs at a speed of $7.7 \mathrm{~m} / \mathrm{s}$. His younger brother runs at $6.5 \mathrm{~m} / \mathrm{s}$. The total length of the race is 50 m .
a) Write an equation to find how long it will take Andrei to finish the race. Solve the equation to find the time.
b) Write an equation to find how long it will take Andrei's younger brother to finish the race. Solve the equation to find the time.
c) Who wins the race? How far ahead was the winner when he crossed the finish line?

