The Real Number System

N-RN

Common Core Cluster

Extend the properties of exponents to rational exponents.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of 5 because we want* $\left(5^{\frac{1}{3}}\right)^3 = 5^{\frac{1}{3}\cdot 3}$ *to hold, so* $\left(5^{\frac{1}{3}}\right)^3$ *must equal 5.*	**N-RN.1** In order to understand the meaning of rational exponents, students can initially investigate them by considering a pattern such as: $2^4 = 16$ $2^2 = 4$ $2^1 = 2$ $2^{\frac{1}{2}} = ?$ What is the pattern for the exponents? They are reduced by a factor of $\frac{1}{2}$ each time. What is the pattern of the simplified values? Each successive value is the square root of the previous value. If we continue this pattern, then $2^{\frac{1}{2}} = \sqrt{2}$. Once the meaning of a rational exponent (with a numerator of 1) is established, students can verify that the properties of integer exponents hold for rational exponents as well. For example, $3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 3^{\frac{1}{2}+\frac{1}{2}} = 3^1 = 3$ since $3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = \sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3$ $\left(5^{\frac{1}{3}}\right)^3 = 5^{\frac{1}{3}\cdot 3} = 5^1 = 5$ since $\left(5^{\frac{1}{3}}\right)^3 = \left(\sqrt[3]{5}\right)^3 = 5$ $\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}} = x^{\frac{1}{5}-\frac{1}{5}} = x^0 = 1$ Ex. Use an example to show why $\frac{x^m}{x^n} = x^{m-n}$ holds true for expressions involving rational exponents like $\frac{1}{2}$ or $\frac{1}{5}$.

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**N-RN.2** Students should be able to use the properties of exponents to rewrite expressions involving radicals as expressions using rational exponents. *At this level, focus on fractional exponents with a numerator of 1.* Ex. Simplify the following. a. $\sqrt[3]{5}\cdot 5^{6}$ b. $x^{\frac{1}{3}}\cdot\sqrt{9x^{6}}$ **N-RN.2** Students should be able to use the properties of exponents to rewrite expressions involving rational exponents as expressions using radicals. *At this level, focus on fractional exponents with a numerator of 1.* Ex. Simplify the following. a. $x^{\frac{1}{4}}\cdot x^{\frac{3}{4}}$ b. $4^{\frac{1}{2}}\cdot 16^{\frac{1}{4}}$

Quantities* N-Q

Common Core Cluster

Reason quantitatively and use units to solve problems.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**N-Q.1** Use units as a tool to help solve multi-step problems. For example, students should use the units assigned to quantities in a problem to help identify which variable they correspond to in a formula. Students should also analyze units to determine which operations to use when solving a problem. Given the speed in mph and time traveled in hours, what is the distance traveled? From looking at the units, we can determine that we must multiply mph times hours to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ (Note that knowledge of the distance formula is not required to determine the need to multiply in this case.) **N-Q.1** Based on the type of quantities represented by variables in a formula, choose the appropriate units to express the variables and interpret the meaning of the units in the context of the relationships that the formula describes. Ex. When finding the area of a circle using the formula $A = \pi r^2$, which unit of measure would be appropriate for the radius? a. square feet b. inches c. cubic yards d. pounds Ex. Based on your answer to the previous question, what units would the area be measured in? **N-Q.1** When given a graph or data display, read and interpret the scale and origin. When creating a graph or data display, choose a scale that is appropriate for viewing the features of a graph or data display. Understand that using larger values for the tick marks on the scale effectively "zooms out" from the graph and choosing smaller values "zooms in." Understand that the viewing window does not necessarily show the x- or y-axis, but the apparent axes are parallel to the x- and y-axes. Hence, the intersection of the apparent axes in the viewing window may not be the origin. Also be aware that apparent intercepts may not correspond to the actual x- or y-intercepts of the graph of a function.

	Ex. What scale would be appropriate for making a histogram of the following data that describes the level of lead in the blood of children (in micrograms per deciliter) who were exposed to lead from their parents' workplace? 10, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 23, 24, 25, 27, 31, 34, 34, 35, 35, 36, 37, 38, 39, 39, 41, 43, 44, 45, 48, 49, 62, 73
N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.	**N-Q.2** Define the appropriate quantities to describe the characteristics of interest for a population. For example, if you want to describe how dangerous the roads are, you may choose to report the number of accidents per year on a particular stretch of interstate. Generally speaking, it would not be appropriate to report the number of exits on that stretch of interstate to describe the level of danger. Ex. What quantities could you use to describe the best city in North Carolina? Ex. What quantities could you use to describe how good a basketball player is?
N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**N-Q.3** Understand that the tool used determines the level of accuracy that can be reported for a measurement. For example, when using a ruler, you can only legitimately report accuracy to the nearest division. If I use a ruler that has centimeter divisions to measure the length of my pencil, I can only report its length to the nearest centimeter. Ex. What is the accuracy of a ruler with 16 divisions per inch?

Seeing Structure in Expressions

A-SSE

Common Core Cluster

Interpret the structure of expressions.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. *b.* Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	**A-SSE.1a**. Students manipulate the terms, factors, and coefficients in difficult expressions to explain the meaning of the individual parts of the expression. Use them to make sense of the multiple factors and terms of the expression. For example, consider the expression $10{,}000(1.055)^5$. This expression can be viewed as the product of 10,000 and 1.055 raised to the 5th power. 10,000 could represent the initial amount of money I have invested in an account. The exponent tells me that I have invested this amount of money for 5 years. The base of 1.055 can be rewritten as (1 + 0.055), revealing the growth rate of 5.5% per year. *At this level, limit to linear expressions, exponential expressions with integer exponents, and quadratic expressions.* Ex. The expression *20(4x) + 500* represents the cost in dollars of the materials and labor needed to build a square fence with side length *x* feet around a playground. Interpret the constants and coefficients of the expression in context. **A-SSE.1b** Students group together parts of an expression to reveal underlying structure. For example, consider the expression $4000p - 250p^2$ that represents income from a concert where *p* is the price per ticket. The equivalent factored form, $p(4000 - 250p)$, shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. *At this level, limit to linear expressions, exponential expressions with integer exponents, and quadratic expressions.* Ex. Without expanding, explain how the expression $4(xy-1)^2 + 3(xy-1) + 1$ can be viewed as having the structure of a quadratic expression.
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	**A.SSE.2** Students rewrite algebraic expressions by combining like terms or factoring to reveal equivalent forms of the same expression. Ex. Expand the expression $2(x-1)^2 - 4$ to show that it is a quadratic expression of the form $ax^2 + bx + c$.

Seeing Structure in Expressions A-SSE

Common Core Cluster

Write expressions in equivalent forms to solve problems.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines.	**A-SSE.3a** Students factor quadratic expressions and find the zeros of the quadratic function they represent. Zeroes are the x-values that yield a y-value of 0. Students should also explain the meaning of the zeros as they relate to the problem. For example, if the expression $x^2 – 4x + 3$ represents the path of a ball that is thrown from one person to another, then the expression $(x – 1)(x – 3)$ represents its equivalent factored form. The zeros of the function, $(x – 1)(x – 3) = y$ would be $x = 1$ and $x = 3$, because an x-value of 1 or 3 would cause the value of the function to equal 0. This also indicates the ball was thrown after 1 second of holding the ball, and caught by the other person 2 seconds later. *At this level, limit to quadratic expressions of the form* $ax^2 + bx + c$. Ex. The expression $3m^2 - 15m$ is the income gathered by promoters of a rock concert based on the ticket price, m. For what value(s) of m would the promoters break even?

Arithmetic with Polynomials and Rational Expressions A-APR

Common Core Cluster

Perform arithmetic operations on polynomials

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**A-APR.1** The Closure Property means that when adding, subtracting or multiplying polynomials, the sum, difference, or product is also a polynomial. Polynomials are not closed under division because in some cases the result is a rational expression rather than a polynomial. *At this level, limit to addition and subtraction of quadratics and multiplication of linear expressions.* **A-APR.1** Add, subtract, and multiply polynomials. *At this level, limit to addition and subtraction of quadratics and multiplication of linear expressions.* Ex. If the radius of a circle is $5x - 2$ kilometers, what would the area of the circle be? Ex. Explain why $(4x + 3)^2$ does not equal $(16x^2 + 9)$.

Creating Equations* A-CED

Common Core Cluster

Create equations that describe numbers or relationships

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	**A-CED.1** From contextual situations, write equations and inequalities in one variable and use them to solve problems. Include linear and exponential functions. *At this level, focus on linear and exponential functions.* Ex. The Tindell household contains three people of different generations. The total of the ages of the three family members is 85. a. Find reasonable ages for the three Tindells. b. Find another reasonable set of ages for them. c. One student, in solving this problem, wrote $C + (C+20) + (C+56) = 85$ 1. What does C represent in this equation? 2. What do you think the student had in mind when using the numbers 20 and 56? 3. What set of ages do you think the student came up with? Ex. A salesperson earns $700 per month plus 20% of sales. Write an equation to find the minimum amount of sales needed to receive a salary of at least $2500 per month. Ex. A scientist has 100 grams of a radioactive substance. Half of it decays every hour. Write an equation to find how long it takes until 25 grams are left.
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	**A-CED.2** Given a contextual situation, write equations in two variables that represent the relationship that exists between the quantities. Also graph the equation with appropriate labels and scales. Make sure students are exposed to a variety of equations arising from the functions they have studied. *At this level, focus on linear, exponential and quadratic equations. Limit to situations that involve evaluating exponential functions for integer inputs.* Ex. In a woman's professional tennis tournament, the money a player wins depends on her finishing place in the standings. The first-place finisher wins half of $1,500,000 in total prize money. The second-place finisher wins half of what is left; then the third-place finisher wins half of that, and so on. a. Write a rule to calculate the actual prize money in dollars won by the player finishing in nth place, for any positive integer n. b. Graph the relationship that exists between the first 10 finishers and the prize money in dollars. **c.** What pattern do you notice in the graph? What type of relationship exists between the two variables?

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**A-CED.3** Use constraints which are situations that are restricted to develop equations and inequalities and systems of equations or inequalities. Describe the solutions in context and explain why any particular one would be the optimal solution. *Limit to linear equations and inequalities.* Ex. The Elite Dance Studio budgets a maximum of $100 per month for newspaper and yellow pages advertising. The news paper charges $50 per ad and requires at least four ads per month. The phone company charges $100 dollars for half a page and requires a minimum of two advertisements per month. It is estimated that each newspaper ad reaches 8000 people and that each half page of yellow page advertisement reaches 15,000 people. What combination of newspaper and yellow page advertising should the Elite Dance Studio use in order to reach the maximum number of people? What assumptions did you make in solving this problem? How realistic do you think they are?
A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	**A-CED.4** Solve multi-variable formulas or literal equations, for a specific variable. Explicitly connect this to the process of solving equations using inverse operations. *Limit to formulas which are linear in the variable of interest or to formulas involving squared or cubed variables.* Ex. If $H = \frac{kA(T_1 - T_2)}{L}$, solve for T_2

Reasoning with Equations and Inequalities A-REI

Common Core Cluster

Understanding solving equations as a process of reasoning and explain the reasoning

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?

<table>
<tr>
<td>A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
<td>A-REI.1 Relate the concept of equality to the concrete representation of the balance of two equal quantities. Properties of equality are ways of transforming equations while still maintaining equality/balance. Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process with mathematical properties.

Ex. Solve 5(x+3)-3x=55 for x. Use mathematical properties to justify each step in the process.</td>
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<tr>
<td colspan="2">Common Core Cluster</td>
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<tr>
<td colspan="2">Solve equations and equalities in one variable.</td>
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<tr>
<td>Common Core Standard</td>
<td>Unpacking
What does this standard mean that a student will know and be able to do?</td>
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<tr>
<td>A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
<td>A-REI.3 Solve linear equations in one variable, including coefficients represented by letters.
Ex. Solve, Ax +B =C for x. What are the specific restrictions on A?
Ex. What is the difference between solving an equation and simplifying an expression?
Ex. Grandma's house is 20 miles away and Johnny wants to know how long it will take to get there using various modes of transportation.
a. Model this situation with an equation where time is a function of rate in miles per hour.
b. For each mode of transportation listed below, determine the time it would take to get to Grandma's.
<table>
<tr><th>Mode of Transportation</th><th>Rate of Travel in mph</th><th>Time of Travel hrs.</th></tr>
<tr><td>bike</td><td>12mph</td><td></td></tr>
<tr><td>car</td><td>55mph</td><td></td></tr>
<tr><td>walking</td><td>4mph</td><td></td></tr>
</table>

A-REI.3 Solve linear inequalities in one variable, including coefficients represented by letters.
Ex. A parking garage charges $1 for the first half-hour and $0.60 for each additional half-hour or portion thereof. If you have only $6.00 in cash, write an inequality and solve it to find how long you can park.

Ex. Compare solving an inequality in one variable to solving an equation in one variable, also compare solving a linear inequality to solving a linear equation.</td>
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Common Core Cluster

Solve systems of equations.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**A.REI.6** Solve systems of equations exactly by using the substitution method and solve systems of equations by using the elimination method (sometimes called linear combinations). Ex. Solve the system by elimination, checking your solution by graphing using technology. $3x + 2y = 6$ $x - 4y = 2$ Ex. Solve the system by substitution, checking your solution by graphing using technology. $-3x + 5y = 6$ $2x + y = 6$ **A.REI.6** Solve systems of equations approximately by using graphs. Graph the system of linear functions on the same coordinate plane and find the point of intersection. This point is the solution to the system because it is the one point that makes all equations in the system true. Equations may be in standard or slope-intercept form. Ex. The equations $y = 18 + .4m$ and $y = 11.2 + .54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately. a. Graph each equation on the same set of axes. b. What mass makes the springs the same length? c. What is the length at that mass? d. Write a sentence comparing the two springs.

Common Core Cluster

Represent and solve equations and inequalities graphically.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**A-REI.10** Understand that all points on the graph of a two-variable equation are solutions because when substituted into the equation, they make the equation true. *At this level, focus on linear and exponential equations* Ex. Which of the following points are on the graph of the equation $-5x + 2y = 20$? How many points are on this graph? Explain. a. (4, 0) b. (0, 10) c. (-1, 7.5) d. (2.3, 5)
A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	**A-REI.11** Understand that solving a one-variable equation of the form f(x) = g(x) is the same as solving the two-variable system y = f(x) and y = g(x). When solving by graphing, the x-value(s) of the intersection point(s) of *y = f(x)* and *y = g(x)* is the solution of *f(x) = g(x)* for any combination of linear and exponential functions. Use technology, entering f(x) in y_1 and g(x) in y_2, graphing the equations to find their point of equality. *At this level, focus on linear and exponential functions.* Ex. How do you find the solution to an equation graphically? **A-REI.11** Solve graphically, finding approximate solutions using technology. *At this level, focus on linear and exponential functions.* Ex. Solve the following equations by graphing. Give your answer to the nearest tenth. $3(2^x) = 6x - 7$ $10x + 5 = -x + 8$ **A-REI.11** Solve by making tables for each side of the equation. Use the results from substituting previous values of x to decide whether to try a larger or smaller value of x to find where the two sides are equal. The x-value that makes the two sides equal is the solution to the equation. *At this level, focus on linear and exponential functions.* Ex. Solve the following equations by using a table. Give your answer to the nearest tenth. $\mathbf{3(x + 2) - 8x = 4x + 1}$ $\mathbf{3.5^x = 22}$

A-REI.12 Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**A-REI.12** Understand that all points on a half-plane are solutions to a linear inequality. Ex. How do we use a graph to represent the solutions to a linear inequality? Why do we use a graph instead of listing the solutions (as we do when solving equations)? **A-REI.12** Determine whether the boundary line should be included as part of the solution set. Ex. Decide whether the boundary line should be included for the following inequalities. How many solutions does each inequality have? $3x - 4y \leq 7$ $y > -2x + 6$ $-9x + 4y \geq 1$ **A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for non-inclusive inequalities. Ex. Graph the following inequalities: $3x - 4y \leq 7$ $y > -2x + 6$ $-9x + 4y \geq 1$ **A-REI.12** Understand that the solutions to a system of inequalities in two-variables are the points that lie in the intersection of the corresponding half-planes. Ex. Compare the solution to a system of equations to the solution of a system of inequalities. Ex. Describe the solution set of a system of inequalities. **A-REI.12** Graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding half-planes. Ex. Graph the solution set for the following system of inequalities: $3x + 5y \leq 10$ $y > -4$

Interpreting Functions F-IF

Common Core Cluster

Understand the Concept of a Function and use Function Notation.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input x. The graph of f is the graph of the equation *y* = *f(x)*.	**F-IF.1** The domain of a function is the set of all x-values, which you control and therefore is called the independent variable. The range of a function is the set of all y- values and is dependent on a particular x-value, thus called the dependent variable. *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions should not occur at this level. Students will apply these concepts throughout their future mathematics courses.* Ex. When is an equation a function? Explain the notation that defines a function. Ex. Describe the domain and range of a function and compare the concept of domain and range as it relates to a function.
F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**F-IF.2 Using function notation, evaluate functions and explain values based on the context in which they are in.** *At this level, focus on linear and exponential functions.* Ex. Evaluate $f(2)$ for the function $f(x) = \frac{x-5}{2x}$. Ex. The function $h(x) = -16x^2 + 35x + 6$ describes the height h in feet of a tennis ball x seconds after it is shot straight up into the air from a pitching machine. Evaluate $h(2.5)$ and interpret the meaning of the point in the context of the problem.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	**F-IF.3** A sequence can be thought of as a function, with the input numbers consisting of the integers, and the output numbers being the terms of the sequence. *Connect to arithmetic and geometric sequences (F-BF.2). Emphasize that arithmetic and geometric sequences are examples of linear and exponential functions.* In an arithmetic sequence, each term is obtained from the previous term by adding the same number each time. This number is called the common difference. In a geometric sequence, each term is obtained from the previous term by multiplying by a constant amount, called the common ratio. NOW-NEXT equations are equations that show how to calculate the value of the next term in a sequence from the value of the current term. The arithmetic sequence $NEXT = NOW \pm C$ is the recursive form of a linear function. The common difference C corresponds to the slope m in the explicit form of a linear function, $y = mx + b$. The initial value of the sequence corresponds to the y-intercept, b. The geometric sequence $NEXT = B \bullet NOW$ is the recursive form of an exponential function. The common ratio B corresponds to the base b in the explicit form of an exponential function, $y = ab^x$. The initial value corresponds to the y-intercept, a. NOW-NEXT equations are the first step in the process of formalizing a sequence using function notation. The NOW-NEXT representation allows students to explore and understand the concept of a recursive function before being introduced to symbolic notation such as $A_n = A_{n-1} + 6$ or $f(n) = f(n-1) + 6$. *In the first course, students need only convert between recursive and explicit forms for arithmetic and geometric sequences.* Ex. You just got a pair of baby rabbits for your birthday – one male and one female. You decide that you will breed the rabbits, but need to plan a budget for the upcoming year. To help prepare your budget, you need an estimate of how many rabbits you will have by the end of the year. In order to build a mathematical model of this situation, you make the following assumptions: • A rabbit will reach sexual maturity after one month. • The gestation period of a rabbit is one month. • Once a female rabbit reaches sexual maturity, she will give birth every month. • A female rabbit will always give birth to one male rabbit and one female rabbit. • Rabbits never die. So how many male/female rabbit pairs are there after one year (12 months)? Ex. In August 2011, the population in the United States was approximately 312 million. Suppose that in recent trends the birth rate was 1.7% of the total population. Use the word NOW to represent the population of the United

States in any given year and the word NEXT to represent the population the following year. Using NOW-NEXT, write a rule that shows how to calculate next year's population from the current population. What is the initial value of this sequence?

Ex. A single bacterium is placed in a test tube and splits in two after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues for one hour until test tube is filled up. How many bacteria are in the test tube after 5 minutes? 15 minutes? Write a recursive rule to find the number of bacteria in the test tube after *n* minutes. Convert this rule into explicit form. How many bacteria are in the test tube after one hour?

Common Core Cluster

Interpret functions that arise in applications in terms of the context.

Common Core Standard

Unpacking

What does this standard mean that a student will know and be able to do?

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.4 When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the graph in the context of the problem. The characteristics described should include rate of change, intercepts, maximums/minimums, symmetries, and intervals of increase and/or decrease. *At this level, focus on linear, exponential, and quadratic functions; no end behavior or periodicity.*

Ex. Below is a table that represents the relationship between daily profit, P for an amusement park and the number of paying visitors in thousands, n.

n	P
0	0
1	5
2	8
3	9
4	8
5	5
6	0

a. What are the x-intercepts and y-intercepts and explain them in the context of the problem.
b. Identify any maximums or minimums and explain their meaning in the context of the problem.
c. Determine if the graph is symmetrical and identify which shape this pattern of change develops.
d. Describe the intervals of increase and decrease and explain them in the context of the problem.

Ex. A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h(t) = – 16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet.

a. What is the practical domain for t in this context? Why?
b. What is the height of the rocket two seconds after it was launched?
c. What is the maximum value of the function and what does it mean in context?
d. When is the rocket 100 feet above the ground?
e. When is the rocket 250 feet above the ground?
f. Why are there two answers to part e but only one practical answer for part d?
g. What are the intercepts of this function? What do they mean in the context of this problem?
h. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem?

F-IF.4 When given a verbal description of the relationship between two quantities, sketch a graph of the relationship, showing key features.

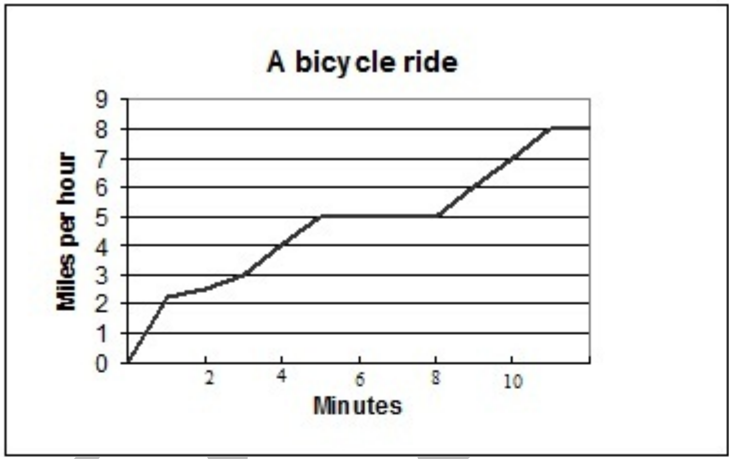
Ex. Elizabeth and Joshua tried to get a monthly allowance from their mother. If their mother initially paid them a penny and 2 pennies for the first day of the month, 4 pennies for the second day, and so on. How much would their mother have to pay on the 10th, 20th, and 30th day of the month? Sketch the graph of the relationship between the two quantities and explain what the point (0, 1) represents.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F- IF.5
From a graph students will identify the domain. In context, students will identify the domain, stating any restrictions and why they are restrictions. *At this level, focus on linear and exponential functions.*

Ex. If Jennifer buys a cell phone and the plan she decided upon charged her $50 for the phone and $0.10 for each minute she is on the phone. What would be the appropriate domain that describes this relationship? Describe what is meant by the point (10, 51).

Ex. Graph the function $f(x) = 4^x + 7$ and determine the domain and range, identifying any restrictions on that exist.

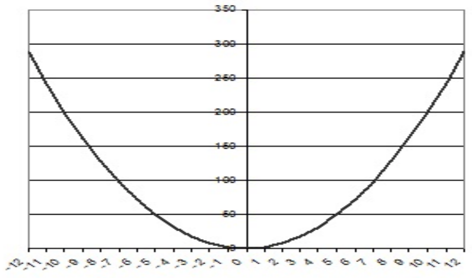
F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	**F-IF.6** Students should find the average rate of change of a function if given a table, an equation, or a graph. The average rate of change over the interval [a, b] is the ratio of the change in y-values to the change in x-values. *i.e.* $\frac{\nabla y}{\nabla x} = \frac{f(b)-f(a)}{b-a}$ Interpret what the average rate of change means in terms of the context it is in. *At this level, focus on linear functions and exponential functions whose domain is a subset of the integers.* Ex. What is the average rate at which this bicycle rider traveled from four to ten minutes of her ride? 

Common Core Cluster

Analyze functions using different representations.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic	**F-IF.7** Students should graph functions given by an equation and show characteristics such as but not limited to intercepts, maximums, minimums, and intervals of increase or decrease. Students may use calculators or a CAS for more difficult cases. *At this level, for part e, focus on exponential functions only.* Ex. Graph $f(x) = -4.9t^2 + 20t$, identifying it's intercepts and maximum or minimum. Ex. Graph $f(x) = 2^{x+1}$, identifying it's intercepts.

functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. *b.* Use the properties of exponents to interpret expressions for exponential	**F-IF.8** Students should take a function and manipulate it in a different form so that they can show and explain special properties of the function such as; zeros, extreme values, and symmetries. Students should factor and complete the square to find special properties and interpret them in the context of the problem. Keep in mind when completing the square, the coefficient on the x^2 variable must always be one and what you add in to the problem, you must also subtract from the problem. In other words, we are adding zero to the problem in order to manipulate it and get it in the form we want. *At this level, only factoring expressions of the form $ax^2 + bx + c$, is expected. Completing the square is not addressed.* Ex. Suppose you have a rectangular flower bed whose area is 24ft^2. The shortest side is (x-4)ft and the longest side is (2x)ft. Find the length of the shortest side. Ex. The Falling Freely Skydiving Company charges a basic price of $150 per person for each jump. However, business is slow and to attract more clients, the company reduces the price of each jump by $5 for each person in the group. The larger the group, the less each person pays. a. Define variables and write an equation for the price of a single jump. b. If you and a group of your friends decided to go skydiving, what would the equation be for the total price the company charges? c. What is the total price of a jump for a group of 6 people? d. The company reports that the cost of the skydiving trip was $1000.00, How many people were on the trip? e. What limitations on group size should the skydiving company use in order to make a profit?

functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	Ex. Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. a. Write an equation for the situation above. b. How do you know you have the correct equation? Justify you reasoning. c. After how many hours will there be at least 100,000 bacteria present in the new colony? d. What is the rate of change for this situation? Does the rate represent growth or decay?
F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**F-IF.9** Students should compare the properties of two functions represented by verbal descriptions, tables, graphs, and equations. *For example, compare the growth of two linear functions, two exponential functions, or one of each. At this level, limit to linear, exponential, and quadratic functions.* Ex. Compare the functions represented below. Which has the lowest minimum? a. $f(x) = 3x^2 + 13x + 4$ b.  Ex. Compare the patterns of (x, y) values when produced by these functions: $f(x) = 2(3^x)$and $g(x) = 2 + 3x$ by completing these tasks. a. Write a NOW- NEXT equation that would provide the same pattern of (x, y) values for each function. b. How would you describe the similarities and differences in the relationships of x and y in terms of their graphs, tables, and equations.

Building Functions F-BF

Common Core Cluster

Build a Function that Models a Relationship Between two Quantities

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
F-BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	**F-BF.1a** Recognize when a relationship exists between two quantities and write a function to describe them. Use steps, the recursive process, to make the calculations from context in order to write the explicit expression that represents the relationship. Ex. A single bacterium is placed in a test tube and splits in two after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues for one hour until test tube is filled up. How many bacteria are in the test tube after 5 minutes? 15 minutes? Write a recursive rule to find the number of bacteria in the test tube after *n* minutes. Convert this rule into explicit form. How many bacteria are in the test tube after one hour? **F-BF.1b** Students should take standard function types such as constant, linear and exponential functions and add, subtract, multiply and divide them. Also explain how the function is effected and how it relates to the model. *At this level, limit to addition or subtraction of a constant function to linear, exponential, or quadratic functions or addition of linear functions to linear or quadratic functions.* Ex. Suppose Kevin had $10,000 to invest in a CD account paying 8% interest compounded yearly. The function representing this situation is $y = 10000(1.08^x)$. When the constant function y=50 is added to the function, what effect does it have on the exponential function? What does that mean in the context of the problem?

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	**F-BF.2** Write the recursive and explicit forms of the arithmetic and geometric sequences. Translate between the recursive and explicit forms. Use the recursive and explicit forms of arithmetic and geometric sequences to model real-world situations. In an arithmetic sequence, each term is obtained from the previous term by adding the same number each time. This number is called the common difference. In a geometric sequence, each term is obtained from the previous term by multiplying by a constant amount, called the common ratio. *Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. At this level, formal recursive notation is not used. Instead, use of informal recursive notation (such as NEXT = NOW + 5, starting at 3) is intended.* Ex. A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats. Write a recursive formula to find the number of seats in each row. How many seats are in row 5? Write the explicit formula to determine which row has 94 seats?
F-BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	**F-BF.3** Know that when adding a constant, k, to a function, it moves the graph of the function vertically. If k is positive, it translates the graph up, and if k is negative, it translates the graph down. If k is either added or subtracted from the x-value, it translates the graph of the function horizontally. If we add k, the graph shifts left and if we subtract k, the graph shifts right. The expression (x + k) shifts the graphs k units to the left because when x + k = 0, x = -k. Us the calculator to explore the effects these values have when applied to a function and explain why the values affect the function the way it does. The calculator visually displays the function and its translation making it simple for every student to describe and understand translations. *At this level, limit to vertical and horizontal translations of linear and exponential functions. Even and odd functions are not addressed. Relate the vertical translation of a linear function to its y-intercept.* Ex. Given $g(x) = x^2$ describe the changes in the graph of g(x) , that occurred to create $f(x) = 2(x – 5)^2 + 7$. Ex. If f(x) represents a diver's position from the edge of a pool as he dives from a 5ft. long board 25ft. above the water. If his second dive was from a 10ft. long board that is 10ft above the water, what happens to my equation of f(x) to model the second dive?

Linear and Exponential Models★ F-LE

Common Core Cluster

Construct and compare linear and exponential models and solve problems.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	**F-LE.1** Decide whether a situation can be represented using a linear model or an exponential model. Recursive forms of functions will show that linear models grow by a constant rate over equal intervals, and exponential models grow by equal factors over equal intervals. These can also be seen through NOW-NEXT equations. NOW-NEXT equations are equations that show how to calculate the value of the next term in a sequence from the value of the current term. The arithmetic sequence $NEXT = NOW \pm C$ is the recursive form of a linear function. The common difference C corresponds to the slope m in the explicit form of a linear function, $y = mx + b$. The initial value of the sequence corresponds to the y-intercept, b. The geometric sequence $NEXT = B \bullet NOW$ is the recursive form of an exponential function. The common ratio B corresponds to the base b in the explicit form of an exponential function, $y = ab^x$. The initial value corresponds to the y-intercept, a. Ex. If one person does good deeds for three new people, then the three new people each do good deeds for three more new people. Next, nine people each do good deeds for three more new people, and so on. Does this situation represent a linear or exponential model? Why or why not? Ex. Describe the similarities and differences of a linear and an exponential NOW- NEXT equation. **F-LE.1a** Given a linear function, use a table or graph to locate points that are at equal intervals of x. Calculate the difference between values of f(x), showing that these differences are the same. Hence linear functions grow by equal differences over equal intervals. **F-LE.1a** Given an exponential function, use a table or graph to locate points that are at equal intervals of x. Calculate the ratio of the sequential values of f(x) for these points, showing that these ratios are the same. This ratio represents the constant factor that is used as the base of the function. In NOW-NEXT form, the base is the factor that is multiplied by NOW to get to NEXT value. Hence exponential functions grow by equal factors over equal intervals.

Ex. Describe how you decide whether a problem situation is represented by a linear or an exponential model?

F-LE.1b Given a real-life relationship between two quantities, determine whether one quantity changes at a constant rate per unit interval of the other quantity. When working with symbolic form of the relationship, if the equation can be rewritten in the form y = mx + b, then the relationship is linear and the constant rate per unit interval is m. When working with a table or graph, either write the corresponding equation and see if it is linear or locate at least two pairs of points and calculate the rate of change for each set of points. If these rates are the same, the function is linear. If the rates are not all the same, the function is not linear.

Ex. Describe and compare the rates of change of a linear function to that of an exponential function.

F-LE.1c Given a real-life relationship between two quantities, determine whether one quantity changes at a constant percent rate per unit interval of the other quantity. When working with symbolic form of the relationship, if the equation can be rewritten in the form $y = a(1 + r)^x$, then the relationship is exponential and the constant percent rate per unit interval is r. When working with a table or graph, either write the corresponding equation and see if it is exponential or locate at least two pairs of points and calculate the percent rate of change for each set of points. If these percent rates are the same, the function is exponential. If the percent rates are not all the same, the function is not exponential.

Ex. Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain. Report the constant rate per unit interval (linear) or the constant percent rate per unit interval (exponential).

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.2 Students use graphs, a verbal description, two-points(in the linear case), and reading from a table of values to write the linear or exponential function that each of these representations describes. Also include arithmetic and geometric sequences because an arithmetic sequence is linear in pattern and a geometric sequence has an exponential pattern.

Ex. Suppose a single bacterium lands in a cut on your hand. It begins spreading an infection by growing and splitting into two bacteria every 10 minutes. The table below represents the number of bacteria in the cut after several 10-minute intervals.

Number of 10-minute periods	1	2	3	4	5	6	7
Bacteria Count	2	4	8	16	32	64	128

a. Use NOW-NEXT to write a rule relating the number of bacteria at one time to the number 10 minutes later.
b. Write an equation showing how the number of bacteria can be calculated from the number of stages in the growth and division process.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.3 When students compare graphs of various functions, such as linear, exponential, quadratic, and polynomial they should see that any values that increase exponentially eventually increases or grows at a faster rate than values that increase linearly, quadratically, or any polynomial function. *At this level, limit to linear, exponential, and quadratic functions; general polynomial functions are not addressed.*

Ex. Carrie and Elizabeth applied for a job at the local seafood market. Carrie asked for $2 and hour, but the boss proposed giving them $.010 for the first hour, $.020 for the second hour, $.040 for the third hour, $.080, and so on. Below is a table of the hours worked per week and the pay for each hour for each of the two pay plans. When would you want to use Carrie's plan and when would you use the Boss's plan? Why?

Hours worked in a week	1	2	3	4	5	6	7	8	9	10
Earnings for Carrie's Plan	2	4	6	8	10	12	14	16	18	20
Earnings for the Boss's Plan	0.10	0.30	0.70	1.50	3.10	6.30	12.70	25.50	51.10	102.30

Common Core Cluster

Interpret expressions for functions in terms of the situation they model.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	**F-LE.5** Understand the difference between the practical and the non-practical domain in linear and exponential situations and explain their meaning in terms of their context. Identify any values for which the exponential function may approach but does not reach and interpret its meaning in terms of the context it's in. *You may want to talk about asymptotes here, which will be a connection with simple rational functions.* Ex. A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What does it mean in terms of the savings account? What is the meaning of the constant P in terms of the savings account? Explain your reasoning. Will n or f(n) ever take on the value 0? Why or why not?

Congruence

Common Core Cluster

Experiment with transformation in the plane.

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**G-CO.1** Know that a point has position, no thickness or distance. A line is made of infinitely many points, and a line segment is a subset of the points on a line with endpoints. A ray is defined as having a point on one end and a continuing line on the other. An angle is determined by the intersection of two rays. A circle is the set of infinitely many points that are the same distance from the center forming a circular are, measuring 360 degrees. Perpendicular lines are lines in the interest at a point to form right angles. Parallel lines that lie in the same plane and are lines in which every point is equidistant from the corresponding point on the other line. Ex. How would you determine whether two lines are parallel or perpendicular?

Expressing Geometric Properties with Equations G-GPE

Common Core Cluster

Use Coordinates to Prove Simple Geometric Theorems Algebraically

Common Core Standard	**Unpacking**

	What does this standard mean that a student will know and be able to do?
G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).	**G-GPE.4** Use the concepts of slope and distance to prove that a figure in the coordinate system is a special geometric shape. Ex. The coordinates are for a quadrilateral, (3, 0), (1, 3), (-2, 1), and (0,-2). Determine the type of quadrilateral made by connecting these four points? Identify the properties used to determine your classification. You must give confirming information about the polygon. . Ex. If Quadrilateral ABCD is a rectangle, where A(1, 2), B(6, 0), C(10,10) and D(?, ?) is unknown. a. Find the coordinates of the fourth vertex. b. Verify that ABCD is a rectangle providing evidence related to the sides and angles.
G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**G-GPE.5** Use the formula for the slope of a line to determine whether two lines are parallel or perpinducular. Two lines are parallel if they have the same slope and two lines are perpendicular if their slopes are opposite reciprocals of each other. In other words the product of the slopes of lines that are perpendicular is (-1). Find the equations of lines that are parallel or perpendicular given certain criteria. Ex. Suppose a line k in a coordinate plane has slope $\frac{c}{d}$. a. What is the slope of a line parallel to k? Why must this be the case? b. What is the slope of a line perpendicular to k? Why does this seem reasonable? Ex. Two points A(0, -4) , B(2, -1) determines a line, AB. a. What is the equation of the line AB? b. What is the equation of the line perpendicular to AB passing through the point (2,-1)? Ex. There is a situation in which two lines are perpendicular but the product of their slopes is not (-1). Explain the situation in which this happens.
G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**G-GPE.6** Given two points on a line, find the point that divides the segment into an equal number of parts. If finding the mid-point, it is always halfway between the two endpoints. The x-coordinate of the mid-point will be the mean of the x-coordinates of the endpoints and the y-coordinate will be the mean of the y-coordinates of the endpoints. *At this level, focus on finding the midpoint of a segment.* Ex. If you are given the midpoint of a segment and one endpoint. Find the other endpoint. a. midpoint: (6, 2) endpoint: (1, 3)

	b. midpoint: (-1, -2) endpoint: (3.5, -7) Ex. If Jennifer and Jane are best friends. They placed a map of their town on a coordinate grid and found the point at which each of their house lies. If Jennifer's house lies at (9, 7) and Jane's house is at (15, 9) and they wanted to meet in the middle, what are the coordinates of the place they should meet?
G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	**G-GPE.7** Students should find the perimeter of polygons and the area of triangles and rectangles using coordinates on the coordinate plane. Ex. John was visiting three cities that lie on a coordinate grid at (-4, 5), (4, 5), and (-3, -4). If he visited all the cities and ended up where he started, what is the distance in miles he traveled?

Geometric Measurement and Dimension G-GMD

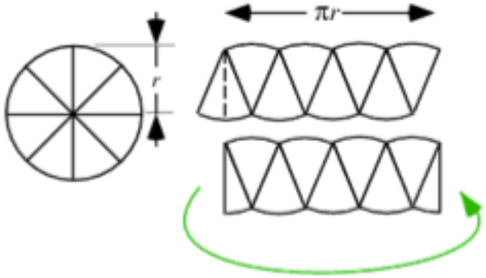
Common Core Cluster

Explain Volume Formulas and Use Them to Solve Problems

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	G-GMD.1 Understanding the formula for the circumference of a circle, you can either begin with the measure of the diameter or the measure of the radius. Take those measurements and measure around the outside of a circle. The diameter will go around a little over 3 times, which indicates $C = \pi d$. The radius will go around half of the circle a little over 3 times, therefore $C = 2\pi r$. This can either be done using pipe cleaners or string and a measuring tool. Understanding the formula for the circumference of a circle can be taught using the diameter of the circle or the radius of the circle. Measure either the radius or diameter with a string or pipe cleaner. As you measure the distance around the circle using the measure of the diameter, you will find that it's a little over three, which is pi. Therefore the circumference can be written as $C = \pi d$. When measuring the circle using the radius, you get a little

over 6, which is 2π. Therefore, the circumference of the circle can also be expressed using $C = 2\pi r$.

Understanding the formula for the area of a circle can be shown using dissection arguments. First dissect portions of the circle like pieces of a pie. Arrange the pieces into a curvy parallelogram as indicated below.



A_{rect} = Base x Height
Area = ½ $(2\pi r)$ x r
Area = πr x r
Area = πr^2

http://mathworld.wolfram.com/Circle.html

Understanding the volume of a cylinder is based on the area of a circle, realizing that the volume is the area of the circle over and over again until you've reached the given height, which is a simplified version of Cavalieri's principle. In Cavalieri's principle, the cross-sections of the cylinder are circles of equal area, which stack to a specific height. Therefore the formula for the volume of a cylinder is $V = Bh$. *Informal limit arguments are not the intent at this level.*

Informal arguments for the volume of a pyramid and cone

Ex.

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

G-GMD.3

Formulas for pyramids, cones, and spheres will be given.

Ex. Given the formula $V = \frac{1}{3}BH$, for the volume of a cone, where B is the area of the base and H is the height of the. If a cone is inside a cylinder with a diameter of 12in. and a height of 16 in., find the volume of the cone.

Interpreting Categorical and Quantitative Data S-ID

Common Core Cluster

Summarize, Represent, and Interpret Data on a Single Count or Measurement Variable.

Common Core Standard

Unpacking
What does this standard mean that a student will know and be able to do?

S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.1 *In grades 6-8, students describe center and spread of a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape (skewed vs. normal) or the presence of outliers.*

Construct appropriate graphical displays (dot plots, histogram, and box plot) to describe sets of data values.

Ex. Make a dot plot of the number of siblings that members of your class have.

Ex. Create a frequency distribution table and histogram for the following set of data:

Age (in months) of First Steps			
13	9	12	11
10	8.5	14	9
12.5	10	13.5	9.5
6	7.5	15	9
8	11.5	10	12
10.5	11	13	12.5

Ex. Construct a box plot of the number of buttons each of your classmates has on their clothing today.

S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.2 Understand which measure of center and which measure of spread is most appropriate to describe a given data set. The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.

Ex. You are planning to take on a part time job as a waiter at a local restaurant. During your interview, the boss told you that their best waitress, Jenni, made an average of $70 a night in tips last week. However, when you asked Jenni about this, she said that she made an average of only $50 per night last week. She provides you with a copy of her nightly tip amounts from last week (see below). Calculate the mean and the median tip amount.

a. Which value is Jenni's boss using to describe the average tip? Why do you think he chose this value?
b. Which value is Jenni using? Why do you think she chose this value?
c. Which value best describes the typical amount of tips per night? Explain why.

Day	Tip Amount
Sunday	$50
Monday	$45
Wednesday	$48
Friday	$125
Saturday	$85

S-ID.2 Select the appropriate measures to describe and compare the center and spread of two or more data sets in context.

Ex. Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 10 days, she measured the heights (in cm) of each set of seedlings. The data she collected is shown below. Construct box plots to analyze the data. Write a brief description comparing the three types of fertilizer. Which fertilizer do you recommend that Delia use?

Fertilizer A		
7.1	6.3	1.0
5.0	4.5	5.2
3.2	4.6	2.4
5.5	3.8	1.5
6.2	6.9	2.6
Fertilizer B		
11.0	9.2	5.6
8.4	7.2	12.1
10.5	14.0	15.3
6.3	8.7	11.3
17.0	13.5	14.2

Fertilizer C		
10.5	11.8	15.5
14.7	11.0	10.8
13.9	12.7	9.9
10.3	10.1	15.8
9.5	13.2	9.7

S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**S-ID.3** Understand and be able to use the context of the data to explain why its distribution takes on a particular shape (e.g. are there real-life limits to the values of the data that force skewness? are there outliers?) Ex. Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right? Ex. Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left? Ex. Why does the shape of the distribution of heights of the students at your school tend to be symmetrical? **S-ID.3** Understand that the higher the value of a measure of variability, the more spread out the data set is. Ex. On last week's math test, Mrs. Smith's class had an average of 83 points with a standard deviation of 8 points. Mr. Tucker's class had an average of 78 points with a standard deviation of 4 points. Which class was more consistent with their test scores? How do you know? **S-ID.3** Explain the effect of any outliers on the shape, center, and spread of the data sets. Ex. Explain the relationship between the mean and the median for a data set that has a few high outliers. What would most likely be the shape of its distribution? Ex. The heights of Washington High School's basketball players are: 5 ft 9in, 5 ft 4in, 5 ft 7 in, 5ft 6 in, 5 ft 5 in, 5 ft 3 in, and 5 ft 7 in. A student transfers to Washington High and joins the basketball team. Her height is 6 ft 10in. a. What is the mean height of the team before the new player transfers in? What is the median height? b. What is the mean height after the new player transfers? What is the median height? c. What affect does her height have on the team's height distribution and stats (center and spread)? d. How many players are taller than the new mean team height? e. Which measure of center most accurately describes the team's average height? Explain.
Common Core Cluster	
Summarize, Represent, and Interpret Data on Two Categorical and Quantitative Variables.	
Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?

S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.5 Create a two-way frequency table from a set of data on two categorical variables.

Ex. Make a two-way frequency table for the following set of data. Use the following age groups: 3-5, 6-8, 9-11, 12-14, 15-17.

Youth Soccer League

Gender	Age	Gender	Age	Gender	Age	Gender	Age	Gender	Age
M	4	F	7	M	17	M	5	F	10
M	7	M	7	M	16	M	9	M	6
F	8	F	15	F	14	F	13	F	4
F	6	M	13	M	14	M	15	M	5
M	4	M	12	F	12	M	17	M	9
F	10	M	15	F	8	M	12	M	10
F	11	F	16	M	13	F	13	F	15

S-ID.5 Calculate joint, marginal, and conditional relative frequencies and interpret in context. Joint relative frequencies are compound probabilities of using AND to combine one possible outcome of each categorical variable (P(A and B)). Marginal relative frequencies are the probabilities for the outcomes of one of the two categorical variables in a two-way table, without considering the other variable. Conditional relative frequencies are the probabilities of one particular outcome of a categorical variable occurring, given that one particular outcome of the other categorical variable has already occurred.

Ex. Use the frequency table to answer the following questions.

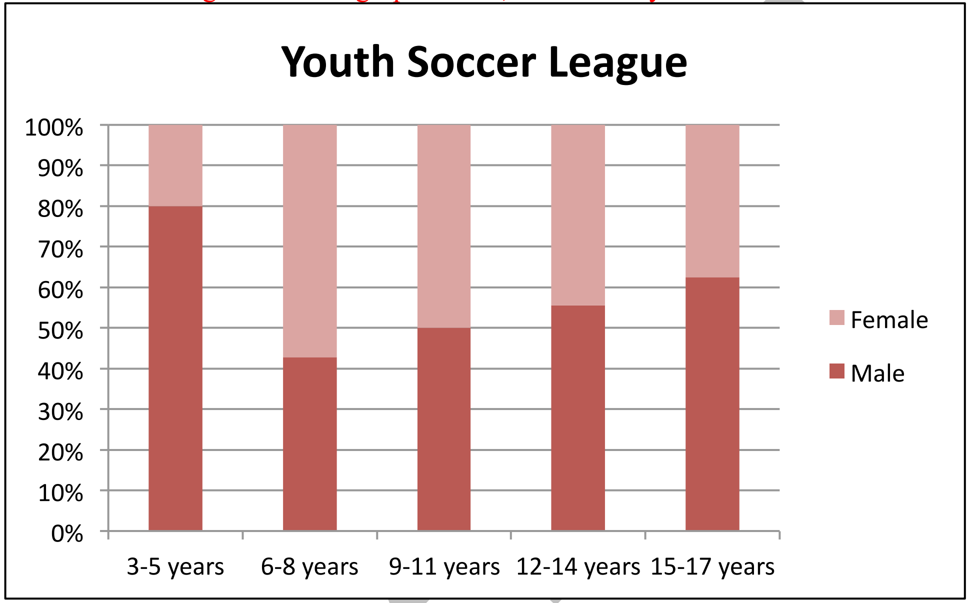
Youth Soccer League

	Age Group					
Gender	**3-5 years old**	**6-8 years old**	**9-11 years old**	**12-14 years old**	**15-17 years old**	**Total**
Male	4	3	3	5	5	**20**
Female	1	4	3	4	3	**15**
Total	5	7	6	9	8	35

a) What is the relative frequency of players who are male and 9-11 years old? (joint relative frequency)
b) What is percentage of female players that are 15-17 years old? (conditional relative frequency)
c) What percentage of league members are male? (marginal relative frequency)

S-ID.5 Recognize associations and trends in data from a two-way table.

Ex. Given the segmented bar graph below, describe any trends in the context of the data.



S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

S-ID.6 Create a scatter plot from two quantitative variables.

Ex. Connie works for a telephone company. She calls existing customers to sell them additional services for their account. The table below shows how much Connie earns for selling selected numbers of additional services. Create a scatter plot of the number of services sold and the daily pay she received.

# of services sold	10	20	30	40	50
Daily Pay in dollars	60	80	100	120	140

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*	**S-ID.6** Describe the form, strength, and direction of the relationship between the two variables in context. Ex. Describe, in context, the form, strength, and direction of the scatterplot from the problem above. **S-ID.6a** Determine which type of function best models a set of data. Fit this type of function to the data and interpret constants and coefficients in the context of the data (e.g. slope and y-intercept of linear models, base/growth or decay rate and y-intercept of exponential models). Use the fitted function to make predictions and solve problems in the context of the data. Ex. What type of function models the data found in the scatterplot above? Find the function that best describes the data. What is the meaning of the slope and y-intercept in the context of the problem? Use the model to predict Connie's earnings for selling 100 services. **S-ID.6b** Calculate the residuals for the data points fitted to a function. A residual is the difference between the actual y-value and the predicted y-value $(y - \hat{y})$, which is a measure of the error in prediction. (Note: $\hat{y}$ is the symbol for the predicted y-value for a given x-value.) A residual is represented on the graph of the data by the vertical distance between a data point and the graph of the function. Ex. Calculate the residuals from the plot above. What do they represent? Are the points with negative residuals located above or below the regression line?
b. Informally assess the fit of a function by plotting and analyzing residuals.	**S-ID.6b** Create and analyze a residual plot. A residual plot is a graph of the x-values vs. their corresponding residuals. (Note that some computer software programs plot $\hat{y}$ vs. residual instead of x vs. residual. However, the interpretation of the residual plot remains the same.) If the residual plot shows a balance between positive and negative residuals and a lack of a pattern, this indicates that the model is a good fit. For more accurate predictions, the size of the residuals should be small relative to the data. *At this level, for part b, focus on linear models.* Ex. What is the sum of the squared residuals of the linear model that represents the situation described above? Can you find a different line that gives a smaller sum? Explain.
c. Fit a linear function for a scatter plot that suggests a linear association.	**S-ID.6c** For data sets that appear to be linear, use algebraic methods and technology to fit a linear function to the data. To develop the concept of LSRL, begin by finding the centroid $(\bar{x}, \bar{y})$ and selecting another point to fit a line through the center of the data. (Note: When describing a set of one-variable data, the mean is the most common predictor of a value in that data set. Therefore, the centroid is a logical choice for a point on the line of best fit because it uses the average of the x-values and the average of the y-values.) Find the sum of the squared errors of this line and compare to lines fitted to the same set of data (but a different second point) by others. The Least Squares Regression Line is a line that goes through the centroid and minimizes the sum of these squared errors.

Ex. Below is the data for the 1919 season and World Series batting averages for nine White Sox players.

Season Batting Average	.319	.279	.275	.290	.351	302	256	282	296
World Series Batting Average	.226	.250	.192	.233	.375	.056	.080	.304	.324

a. Create a scatter plot for the data provided. Is there a linear association? Explain.
b. What is the Least Squares Regression Line that models this data?
c. How do you know this equation is the line of best fit to model the data?

Common Core Cluster

Interpret Linear Models

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**S-ID.7** Understand that the key feature of a linear function is a constant rate of change. Interpret in the context of the data, i.e. as x increases (or decreases) by one unit, y increases (or decreases) by a fixed amount. **S-ID.7** Interpret the y-intercept in the context of the data, i.e. an initial value or a one-time fixed amount. Ex. The equation $C = 40 + 2n$ represents a pay plan offered to employees who collect credit card applications. What do the numbers in the rule tell you about the relationship between daily pay and the number of credit card applications collected?
S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	**S-ID.8** Understand that the correlation coefficient, r, is a measure of the strength and direction of a linear relationship between two quantities in a set of data. The magnitude (absolute value) of r indicates how closely the data points fit a linear pattern. If r = 1, the points all fall on a line. The closer $\|r\|$ is to 1, the stronger the correlation. The closer $\|r\|$ is to zero, the weaker the correlation. The sign of r indicates the direction of the relationship – positive or negative.

Ex. A couple of friends decided to measure their compatibility by ranking their favorite activities.

Mary	4	5	2	7	6	1	3
Maria	7	2	4	3	5	6	1
	Watching TV	Listening to Music	Reading	Talking on the phone	Hanging out with friends	Shopping	Exercise

a. Using technology, make a scatterplot for the two rankings.
b. Predict what the r_s value is. Use the scatterplot to help explain your answer.
c. Find the Least Squares Regression Line that models this set of data.
d. Using technology identify what the correlation coefficient is and interpret what it means in the context of the data.

S-ID.9 Distinguish between correlation and causation.

S-ID.9 Understand that because two quantities have a strong correlation, we cannot assume that the explanatory (independent) variable *causes* a change in the response (dependent) variable. The best method for establishing causation is to conduct an experiment that carefully controls for the effects of lurking variables. If this is not feasible or ethical, causation can be established by a body of evidence collected over time (e.g. smoking causes cancer).

Ex. When you have an association between two variables, how can you determine if the association is a result of a cause-and-effect relationship?